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CHARACTERIZATION OF A RANDOM ANISOTROPIC CONDUCTIVITY FIELD WITH KARHUNEN-LOEVE METHODS (POSTPRINT)

Matthew R. Cherry
University of Dayton Research Institute

Adam L. Pilchak and Jeremy S. Knopp
AFRL/RXCA

Harold S. Sabbagh
Victor Technologies, LLC

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UNITED STATES AIR FORCE

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//Signature//

JOHN T. WELTER, Project Engineer
Materials State Awareness & Supportability Branch
Structural Materials Division

//Signature//

STEPHAN M. RUSS, Chief
Materials State Awareness & Supportability Branch
Structural Materials Division

//Signature//

ROBERT T. MARSHALL, Deputy Chief
Structural Materials Division
Materials and Manufacturing Directorate

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14. ABSTRACT
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6. AUTHOR(S)

Adam L. Pilchak and Jeremy S. Knopp - Materials and Manufacturing Directorate, Air Force Research Laboratory, Structural Materials Division

Matthew R. Cherry - University of Dayton Research Institute

Harold S. Sabbagh - Victor Technologies LLC

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

AFRL/RXCA
Air Force Research Laboratory
Materials and Manufacturing Directorate
Wright-Patterson Air Force Base, OH 45433-7750

University of Dayton Research Institute
Structural Integrity Division
300 College Park Drive
Dayton OH 45469-0020

Victor Technologies, LLC
PO Box 7706
Bloomington IN 47407-7706

Characterization of a Random Anisotropic Conductivity Field with Karhunen-Loeve Methods

Matthew R. Cherry[b], Harold S. Sabbagh[c], Adam L. Pilchak[a], and Jeremy S. Knopp[a]

[a]*Air Force Research Labs, Materials and Manufacturing Directorate*
2230 10th St., WPAFB, OH 45433 USA
[b]*University of Dayton Research Institute, Structural Integrity Division*
300 College Park, Dayton, OH 45469-0020 USA
[c]*Victor Technologies, LLC*
PO Box 7706, Bloomington, IN 47407-7706 USA

Abstract. While parametric uncertainty quantification for NDE models has been addressed in recent years, the problem of stochastic field parameters such as spatially distributed electrical conductivity has only been investigated minimally in the last year. In that work, the authors treated the field as a one-dimensional random process and Karhunen-Loeve methods were used to discretize this process to make it amenable to UQ methods such as ANOVA expansions. In the present work, we will treat the field as a two dimensional random process, and the eigenvalues and eigenfunctions of the integral operator will be determined via Galerkin methods. The Karhunen-Loeve methods is extended to two dimensions and implemented to represent this process. Several different choices for basis functions will be discussed, as well as convergence criteria for each. The methods are applied to correlation functions collected over electron backscatter data from highly micro textured Ti-7Al.

Keywords: Eddy Current, Karhunen, Stochastic, Process
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INTRODUCTION

Uncertainty quantification (UQ) in NDE has been mostly restricted to the development of methods within the context of probability of detection (POD) analysis. Even in the methods of POD, uncertainty in parameters is usually thought of as normally distributed and the statistics models are typically linear in their parameters [1, 2]. The introduction of the notion of model assisted POD (MAPOD) has generated interest in a more thorough understanding of uncertainty and the effect it has on model predictions [3, 4]. In the area of eddy current experiments and simulation, most sources of uncertainty come from variable liftoff, coil tilt, and crack face conditions among other factors. These parameters can be modeled as random variables with known probability density function (PDF) as shown in [5]. However, in a certain class of materials, we find that signals from defects are largely "swamped out" by surrounding material noise. This is a result of inhomogeneous anisotropy in the conductivity constants of the material. At each point in the microstructure, the grain is oriented in an arbitrary direction which changes the effective material constants, electrical conductivity being the relevant property for eddy current testing [6]. A very dramatic example of this is shown in Fig. 1. These images are eddy current scans of the same sized notch in different materials. The first is an isotropic aluminum alloy, whereas the second is an anisotropic titanium alloy (Ti64 β-annealed). It's clear that POD of cracks of this size would be dictated largely by the microstructural features of the alloy.

This issue could be addressed with the methods of image processing. There is a large amount of research in image processing routines and many ready-made algorithms for situations like this. However, if one were to clean up this image, the notch features would still be extracted based on the current practice of estimating damage characteristics based on signal strength and length between peaks. A more physically meaningful result can be attained by means of model based uncertainty quantification (UQ). Model based UQ offers a means of estimating physical parameters with the help of physical models, and furthermore creates an avenue to place confidence intervals on the parameter estimates. Most model based UQ methods require apples-to-apples comparison between the model results and the data. This typically precludes the use of image processing routines since there is often not a direct map from the processed data to the parameters.

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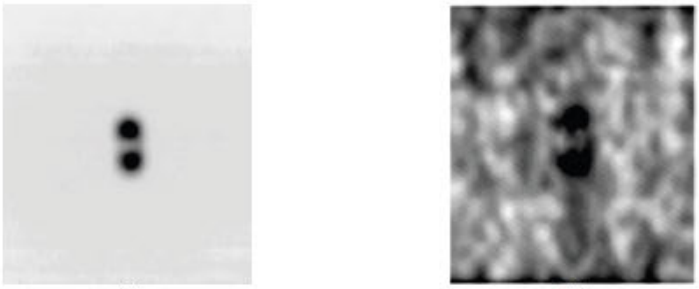


(a) (b)

FIGURE 1. Image of an eddy current scan of a notch in an isotropic material (a) vs. an image of a scan of the same sized notch in an anisotropic material (b). [7]

In this paper, a method of modeling spatially varying random variables will be presented. These types of variables are referred to as a random processes or stochastic processes. Specifically, a random anisotropic conductivity field from a polycrystalline titanium alloy is treated as a random process and statistical methods are used to model the process.

RANDOM PROCESSES

When a parameter is said to be random, we often think of it as a random variable. The random variable can take on any value within a certain range, and each value and subset of values has some probability measure associated to it. The probability measure is the probability density function (PDF) of the random variable. An example of this can be found in mechanical testing, where the measured Young's modulus from different samples of the same material is different for every experiment. This is often reported as a mean and a standard deviation in literature, but this information was arrived at by assuming certain things about the underlying distribution of the Young's modulus. Many parameters in engineering models can be appropriately modeled with this sort of random variable. However, in the case of spatially varying random variables, a new concept called a random process is needed.

Consider a collection of random variables that each correspond to a point in some space (distance, frequency, time, etc.). If a value were picked for each of the random variables according to their distributions, you would be left with what resembles a function over that space. Clearly, if each of these points were picked independently of one another, the function would look like pure noise in that there would be no aggregate trends in the function. If, however, there were some rule governing how the random variables were picked relative to the outcomes of the surrounding random variables, you would start to see this collection of points resemble more of a function over the space. This collection of random variables is called a random process or stochastic process, and the rules governing how the points are picked are first and second order probability density information corresponding to the random process. The first order information is often thought of as a PDF, and the second order information is defined as a correlation function or covariance function. The correlation function dictates that the values in a certain area are controlled, in a sense, by the values in a neighboring area, or in other words, that the random variables at these points are correlated. Many different measures of correlation exist, which indicates that the correlation function can take on many different forms. Stochastic processes will be used to model the anisotropic conductivity over the sample surface of a titanium alloy sample. The specific material will be discussed later.

KARHUNEN-LOEVE METHODS

The goal of the Karhunen-Loeve expansion is to find an expansion that represents the random process so that samples can actually be drawn from the random process. Drawing a sample from random variables with well-defined probability density functions results in a number that lies somewhere in the domain of the random variable. However, it's difficult to imagine drawing a random sample from a distribution of functions because each value that you take is itself a function. The K-L expansion seeks to discretize the random process to a series of deterministic functions and random variables. Sampling from the process then becomes a matter of sampling from random variables and applying the samples in the series expansion.

If we have a random process, $X(\boldsymbol{x}, \Omega)$, where the random variable, X, is a function of the spatial variable $\boldsymbol{x}$, and a random event from Ω, we need to find an expansion of the form:

$$X(\mathbf{x},\Omega) = \sum_{i=1}^{\infty} c_i(\Omega)\varphi_i(\mathbf{x}) \tag{1}$$

In this expansion, each c_i is a random variable with a defined probability density that can be sampled, and each $\varphi_i(\boldsymbol{x})$ is a deterministic function, the form of which to be determined. If this series can be shown to converge to the actual random process, then we have a means of sampling from the process via sampling from the series with an appropriate amount of terms. The functions, $\boldsymbol{\varphi_i}(x)$, turn out to be the eigenfunctions of the correlation function operator:

$$\iint K(\mathbf{x},\mathbf{x}')\varphi_i(\mathbf{x}')d\mathbf{x}' = \lambda_i\varphi_i(\mathbf{x}) \tag{2}$$

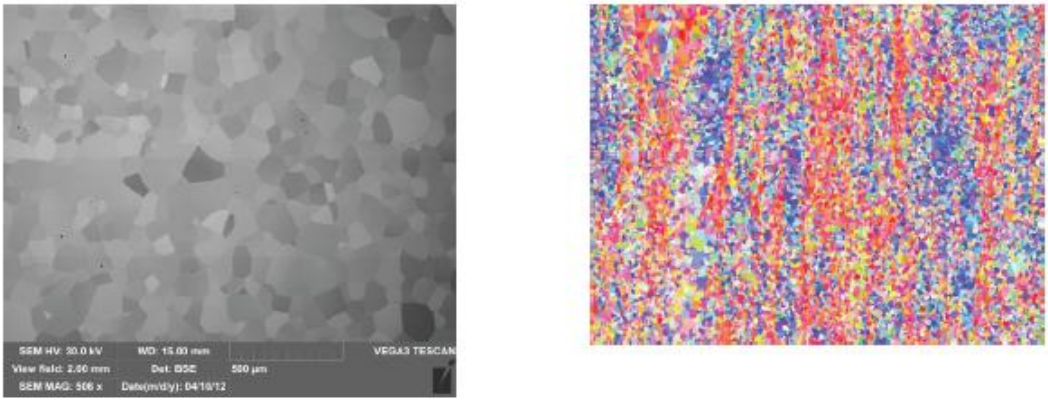
or, in operator notation:

$$L\varphi_i(\mathbf{x}') = \lambda_i\varphi_i(\mathbf{x}) \tag{3}$$

This is an eigenvalue problem with eigenvalues λ_i, and eigenfunctions $\boldsymbol{\varphi_i}(\boldsymbol{x})$. This problem can be solved either analytically if the form of K(x,x') allows, or numerically with methods such as Galerkin's method or the Rayleigh-Ritz method.

MATERIALS

A model hexagonal close packed material was selected for this work. The material, a binary Ti-7wt%Al alloy, has isotropic symmetry in its basal plane, but virtually all physical properties vary as a function of angle from the c-axis. This anisotropy, as discussed earlier, is one of the main sources of noise in eddy current scans. The other major source of noise is related to the length scale of the microstructural constituents relative to the probe size. Titanium alloys are prone to the formation of 'microtexture' in which nominally equiaxed, fully recrystallized grains, such as those in Fig. 2(a), cluster together to form larger regions of similar orientation as illustrated by the crystal orientation map in Fig. 2(b). Here, the color represents the specific $\{hkil\}$ plane perpendicular to the sample normal direction as indicated by the legend. The size and shape of these regions influence fatigue properties as they act as fast paths for crack growth due to the low misorientation boundaries within the microtextured regions. One way to characterize the spatial distribution of similarly oriented grains is through the use of n-point correlation functions [8,9]. The two-point correlation function, specifically the autocorrelation of similarly oriented c-axes for a tolerance angle of 15°, for the dataset in Fig. 2(b) is shown in Fig. 3. The 2-point correlation is elongated which reflects the "streaking" of similarly oriented grains evident in Fig. 2(b).



(a) (b)

FIGURE 2. (a) Micrograph of a Ti-7Al specimen, and (b) an OIM map of a larger region of the specimen.

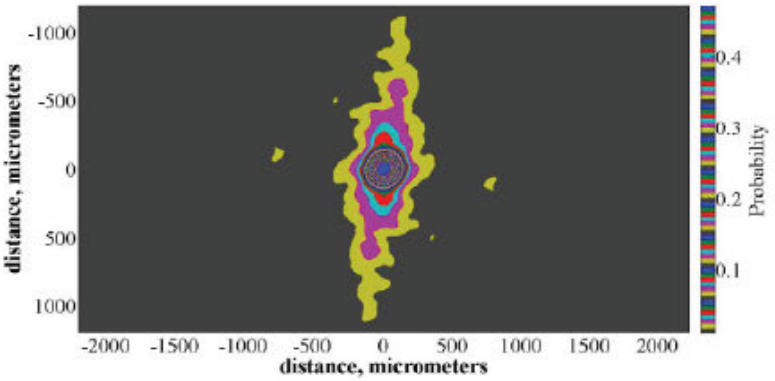


FIGURE 3. Image of the 2-point correlation function calculated from the Ti-7Al microstructure shown

ANALYSIS

The correlation function shown in Fig. 3 is defined numerically and can be used in conjunction with (2) to find its eigenvalues and eigenfunctions. The integral must be performed numerically, which indicates a spatial discretization of the correlation function. As indicated earlier, any Rayleigh-Ritz or Galerkin method will provide a numerical means of obtaining these values. In this case, we choose the Galerkin method with several different choices of basis functions to determine the optimal choice.

Galerkin Method

In the Galerkin method, the eigenvalue problem of (3) is discretized with a set of basis function that satisfy certain properties of the function that is being approximated. In this case, the eigenfunction $\varphi_i(\boldsymbol{x})$ is represented as an expansion in terms of the basis:

$$\varphi_i(\mathbf{x}) = \sum_{k=1}^{\infty} a_k \phi_k(\mathbf{x}) \tag{4}$$

The $\phi_k(\mathbf{x})$ form a set of basis functions and in this context will also be orthogonal to one another. In this problem, there are no continuity restrictions on the basis functions, so one choice of basis could be a set of delta function. Another choice is the standard Fourier basis function $sin(\mathbf{kx})$ and $cos(k\mathbf{x})$. Both of these bases will be tested to determine which converges quicker. Once a basis is chosen, the Galerkin method proceeds by taking the inner product of both sides of (3):

$$\left(L\varphi_i(\mathbf{x}'), \phi_i(\mathbf{x})\right) = \left(\lambda_i \varphi_i(\mathbf{x}), \phi_i(\mathbf{x})\right) \tag{5}$$

or, if (4) is substituted:

$$\left(L\sum_{k=1}^{\infty} a_k \phi_k(\mathbf{x}'), \phi_i(\mathbf{x})\right) = \left(\lambda_i \sum_{k=1}^{\infty} a_k \phi_k(\mathbf{x}), \phi_i(\mathbf{x})\right) \tag{6}$$

Since the basis functions are orthogonal to one another, the expression on the right hand side becomes 0 for $k \neq l$, and (6) reduces to:

$$D\mathbf{a}_i = \lambda_i M\mathbf{a}_i \tag{7}$$

where the elements of D are:

$$d_{lk} = \iint\iint K(\mathbf{x},\mathbf{x}')\varphi_k(\mathbf{x}')\varphi_l(\mathbf{x})d\mathbf{x}d\mathbf{x}' \tag{8}$$

and the M is a diagonal matrix with elements:

$$m_{ll} = \iint \varphi_l^2(\mathbf{x})d\mathbf{x} \tag{9}$$

This is a standard algebraic eigenvalue problem that can be solved with the usual numerical methods. The eigenvalues lambda, are the eigenvalues of the operator, and the eigenvectors, $\boldsymbol{a}_i$, in conjunction with (4) form the corresponding eigenfunctions.

Choice of Basis

In this work, two different choices of basis functions were used to form the algebraic eigenvalue problem in (7) The first is a set of Dirac delta functions that are defined at every point in the correlation function. The second is a set of sine and cosine functions that are defined over the entire domain. The integrations have to be carried ou numerically for this basis choice, which makes it much more costly to use than the Dirac delta basis. Ideally, as basis functions are added, the eigenvalues should taper off to zero. The quicker they decay, the faster the expansion diverges. This will be regarded as the test for convergence for the purpose of this work.

Results

Convergence of the eigenvalues for both choices of basis functions is shown in Fig. 4. It is fairly easy to see tha the delta basis functions converge quicker for this particular correlation function. This coupled with the fact tha analytical solutions exist for the integrals of (8) makes the delta functions a very attractive expansion. Once the eigenvalues and eigenvectors are calculated, the process can be sampled via (1). This assumes that the form of the random variables, $c_i(\Omega)$ is known. For well-defined random processes, this form can often be calculated analytically For instance, these variables are Gaussian for Gaussian random processes. In this case, the form is unknown, but an approximation of uniform was made. This is one of the main areas of improvement that can be made for this work and will be discussed in the future work section. Moving forward with the assumption of normal random variables (1) can be used to directly sample the random process. An example of a sample from this distribution is shown in Fig 5. In this case, the expansion was made with a smaller subset of the correlation function so that direct comparison could be made. What can essentially be seen here is that the spatial variation is on the order of the average grain size of the specimen, which is expected.

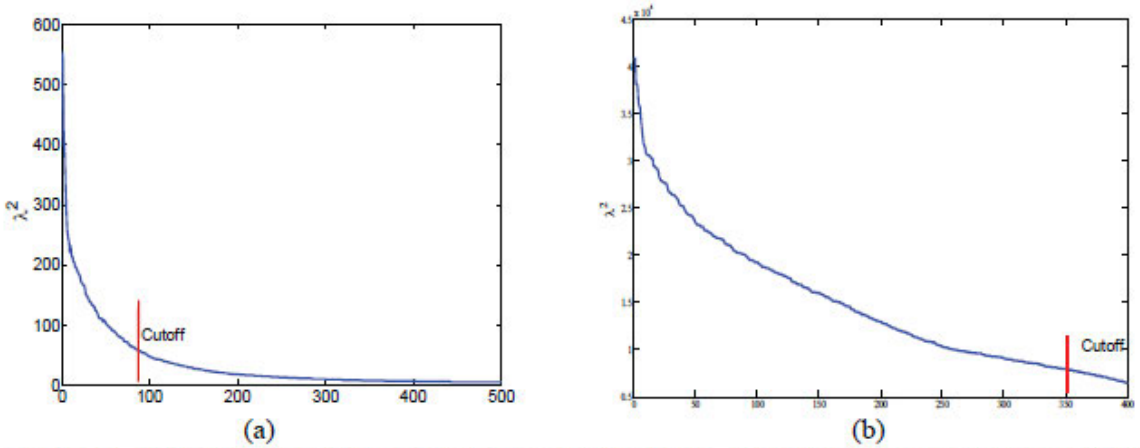


FIGURE 4. Convergence of the (a) Dirac delta basis and the (b) trig basis. If the cutoff is set at 10% maximum, the delta basis clearly converges quicker.

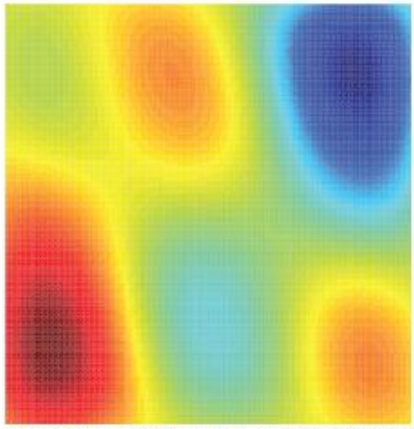
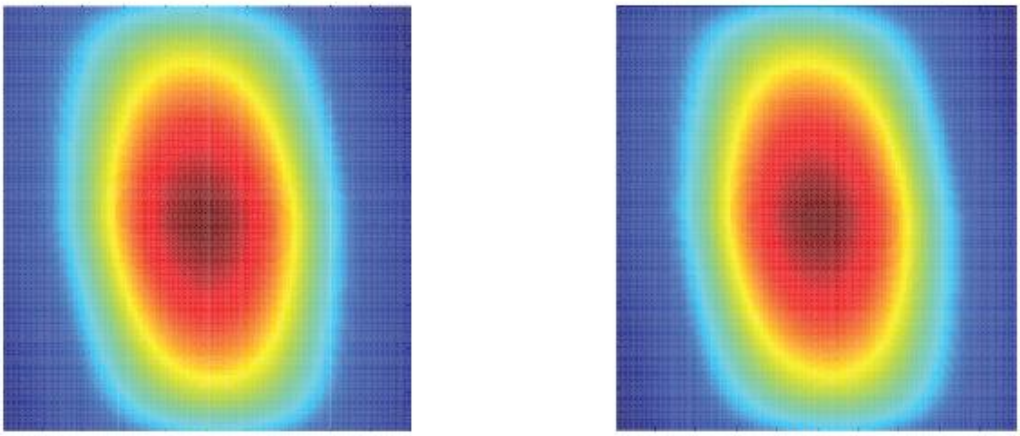


FIGURE 5. Image of a sample from the K-L expansion using delta basis function.

A major method of verification for this expansion is to generate a large amount of samples, and estimate the correlation function. If the expansion has converged and the correct eigenvalues and eigenfunctions are used, this correlation function should correspond to the correlation function that was used to generate the expansion. In other words, you should be able to get back to the information that you started with. An image of the input correlation function vs. the calculated correlation function is shown in Fig. 6. These images are near identical, and indicate that the expansion has indeed converged and is producing results that are expected. The expansion has not been used in the context of a forward uncertainty quantification scheme, but since it is now a series of random variables, it should be relatively straight forward as discussed further below.

DISCUSSION AND FUTURE WORK

The Karhunen-Loeve method was used to discretize a spatially varying 2-dimensional random conductivity field. The method seems to show a promising means of representing a random process for use in the context of UQ schemes. This has not been demonstrated, however. The idea is that instead of a random function, we now have a collection of uncorrelated random variables. Methods for dealing with this situation have been discussed in [5] and use of the K-L expansion for this purpose was discussed in [10]. However, a major concern at this point is that the form of the random variables is unknown, which makes these methods impossible to use. A major goal for future work is to fully characterize the PDF's of these random variables and ensure that they correspond to the random conductivity field.



(a) (b)

FIGURE 6. Images of the correlation function calculated from (a) the sample and (b) the K-L expansion.

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